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Practice Problem for November 17, 2007

May 2007 SOA Course MLC Examination, Problem No. 12
For a special fully discrete 3-year term insurance on (55), whose mortality follows a double decrement model:
(i) Decrement 1 is accidental death; decrement 2 is all other causes of death.
(ii)

x	$q_x^{(1)}$	$q_x^{(2)}$
55	0.002	0.020
56	0.005	0.040
57	0.008	0.060

(iii) $i = 0.06$.
(iv) The death benefit is 2000 for accidental deaths and 1000 for deaths from all other causes.
(v) The level annual contract premium is 50.
(vi) ${}_1L$ is the prospective loss random variable at time 1, based on the contract premium.
(vii) $K(55)$ is the curtate future lifetime of (55).
Calculate the smallest value of z such that $\Pr\left({}_1L \le z \middle| K(55) \ge 1\right) \ge 0.95$.

A. 743 B. 793 C. 843 D. 893 E. 943

Solution.
The expression $K(55) \ge 1$ means that the insured is alive at age 66, and given that situation, there are only five future outcomes of the policy:
• Accidental death at age 56,
• Non-accidental death at age 56,
• Accidental death at age 57,
• Non-accidental death at age 57,
• Survival to age 58.
These five outcomes correspond, respectively, to the following five values of the random variable ${}_1L$, conditional on $K(55) \ge 1$:

- ${}_1L = \dfrac{2000}{1.06} - 50 \approx 1836.7925$ with probability $q_{56}^{(1)} = 0.005$,
- ${}_1L = \dfrac{1000}{1.06} - 50 \approx 893.3962$ with probability $q_{56}^{(2)} = 0.04$,
- ${}_1L = \dfrac{2000}{1.06^2} - 50 - \dfrac{50}{1.06} \approx 1682.8231$ with probability

$$ {}_{1|}q_{56}^{(1)} = p_{56}^{(\tau)} \cdot q_{57}^{(1)} = (1 - 0.005 - 0.04) \cdot 0.008 = 0.00764, $$

- ${}_1L = \dfrac{1000}{1.06^2} - 50 - \dfrac{50}{1.06} \approx 792.8266$ with probability

$$ {}_{1|}q_{56}^{(2)} = p_{56}^{(\tau)} \cdot q_{57}^{(2)} = (1 - 0.005 - 0.04) \cdot 0.06 = 0.0573, $$

- ${}_1L = -50 - \dfrac{50}{1.06} \approx -97.1698$ with probability

$$ {}_2p_{56}^{(\tau)} = p_{56}^{(\tau)} \cdot p_{57}^{(\tau)} = (1 - 0.005 - 0.04) \cdot (1 - 0.008 - 0.06) = 0.89006. $$

We are looking for the smallest value of z such that $\Pr\left({}_1L \le z \middle| K(55) \ge 1\right) \ge 0.95$, i.e., the 95-th percentile of the random variable $\left({}_1L \middle| K(55) \ge 1\right)$. Note that in the above list of values of this random variable, its values are *not* listed in decreasing order, but that order of values is instead

$$ -50 - \frac{50}{1.06} < \frac{1000}{1.06^2} - 50 - \frac{50}{1.06} < \frac{1000}{1.06} - 50 < \frac{2000}{1.06^2} - 50 - \frac{50}{1.06} < \frac{2000}{1.06} - 50. $$

We see that

$$ \Pr\left({}_1L \le -50 - \frac{50}{1.06} \middle| K(55) \ge 1\right) = 0.89006 < 0.95, $$

$$ \Pr\left({}_1L \le \frac{1000}{1.06^2} - 50 - \frac{50}{1.06} \middle| K(55) \ge 1\right) = 0.89006 + 0.0573 = 0.94736 < 0.95, $$

while

$$ \Pr\left({}_1L \le \frac{1000}{1.06} - 50 \middle| K(55) \ge 1\right) = $$

$$ = 0.89006 + 0.0573 + 0.04 = 0.98736 > 0.95. $$

Therefore, the smallest z such that $\Pr\left({}_1L \le z \middle| K(55) \ge 1\right) \ge 0.95$ is

$$ z = \frac{1000}{1.06} - 50 \approx 893.3962. $$

Answer D.